TOM MBOYA UNIVERSITY

FORTH YEAR FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF
EDUCATION

MMA 403: TOPOLOGY

INSTRUCTIONS:

1. This paper consists of FIVE questions

2. Attempt questions ONE and any other TWO questions.

3. Observe further instructions on the answer booklet.

SECTION A

QUESTION 1 **[30 Marks]**

(a) Define the following.

(i) Interior point of a set. **[1 mk]**

(ii) Basis for a topology **[2 mks]**

(b) Let $X = \{a, b\}$. Construct four different topologies on X. **[4 mks]**

(c) Prove that if X is a T_2 space then the sequence $\{x_n\}$ of points in X converges to at most one point in X. **[5 mks]**

(d) Let $f : X \to Y$ and $g : Y \to Z$ be continuous functions. Show that the composition $g \circ f$ is also continuous. **[5 mks]**

(e) Given that $X = \{a, b, c, d, e\}$, and τ is a topology defined on X such that $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\,\{b, c, d, e\}\}$. Let $A \subset X$ such that $A = \{a, b, c\}$.

(i) Find the interior point of A. **[3 mks]**

(ii)Find the limit point of A. **[5 mks]** .

(f) Determine whether or not each of the following intervals is a neighbourhood of the point 0 under the usual topology for the real line $\mathbf{R}$.

(i) $(-1/2, 1/2]$ **[1 mk]**

(ii) $(0, 1]$ **[1 mk]**

(g) Let $X = \{a, b, c, d, e\}$. Define a topology τ on X as $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$. If $A \subset X$ such that $A = \{a, c, e\}$, construct a relative topology on A. **[3 mks]**

QUESTION 2 **[20 Marks]**

(a) Show that for a topological space (X, τ), if $A \subset X$ then $\bar{A} = A \cup A'$ (where A' denotes the limit point of A). **[5 mks]**

(b) Prove that if (X, τ) is a topological space and $A \subset X$, then the interior point of A is the union of all open subsets of A. **[8 mks]**

(c) Let (X, τ_X) and (Y, τ_Y) be two topological spaces. Show that the map $f : X \to Y$ is closed if and only if $\overline{f(A)} \subset f(\bar{A})$ for all $A \subset X$. **[7 mks]**

QUESTION 3 **[20 Marks]**

(a) (i) Define a Homeomorphism **[2 mks]**
(ii) Show that two open intervals $X = (0, 1)$ and $Y = (a, b)$ with $a < b$, each with subspace topology are homeomorphic . **[4 mks]**

(b) Prove that a topological space (X, τ) is a T_1 space if and only if every singleton subset $\{p\} \subset X$ is closed.. **[8 mks]**

(c) Prove that a set $G \subset X$ is open if and if it is a neighbourhood of its points. **[6 mks]**

QUESTION 4 **[20 Marks]**

(a) Let $X = \{a, b, c\}$. Define τ on X as $\tau = \{X, \emptyset, \{a, b\}, \{a, c, \}\}$.Determine whether or not τ is a topology on X. **[2 mks]**

(b) Prove that for a topological space (X, τ), the intersection of any collection of closed subsets of X is closed. **[5 mks]**

(c) (i) Define a Haursdoff space. **[2 mks]**
(ii) Show that every metric space is a Haursdoff space. **[6 mks]**

(d) (i)Differentiate between a closed set A and a closure of the set A in a topological space. **[2 mks]**
(ii) Prove that if A is a subset of B then every limit point of A is also a limit point of B. **[3 mks]**

QUESTION 5 **[20 Marks]**

(a) State the first countability axiom. **[2 mks]**

(b) Prove that every subspace of a $T_{5/2}$ is a $T_{5/2}$ space. **[6 mks]**

(c) Define a metric topology on X hence show that the collection of metric topology is a topology on X. **[12 mks]**