

TOM MBOYA UNIVERSITY

UNIVERSITY EXAMINATION 2023 / 2024

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE
DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

MMA 405: PARTIAL DIFFERENTIAL EQUATIONS

Instructions:

- Answer QUESTION ONE (30 marks) and any other TWO questions (20 marks each).
- Proofs should be written carefully.
- Cheating is NOT allowed and will be harshly punished.
- Observe further instructions on the answer booklet.

QUESTION ONE (Compulsory)

[30 Marks]

- (a) (i) Describe Pfaffian differential equation [2 mks]
(ii) Find the integral curves of the equations

$$\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$$

[5 mks]

- (b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$. [5 mks]
- (c) (i) Eliminate the constants a and b from the equation $z = (x^2 + a)(y^2 + b)$. [3 mks]
(ii) Verify that the equation $(2xyz + z^2)dx + x^2zdy + (xz + 1)dz = 0$ is integrable hence determine its primitive solution. [6 mks]
- (d) Find the integral surface of $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ containing the straight line $x + y = 0, z = 1$. [6 mks]
- (e) Find a complete integral of the equation $\sqrt{p} + \sqrt{q} = 1$. [3 mks]

QUESTION TWO

[20 Marks]

- (a) Form a partial differential equation by eliminating arbitrary functions from the equation $z = f(xy) + g(\frac{x}{y})$ [6 mks]
- (b) Use Natani's method to solve

$$z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$$

[8 mks]

- (c) Find the surface which intersects the surface of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. [6 mks]

QUESTION THREE

[20 Marks]

- (a) Show that the partial differential equations $xp - yq = x$ and $x^2p - q = xz$ are compatible hence find their solutions. [10 mks]
- (b) Let $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ be two first order partial differential equations show that a necessary and sufficient condition for f and g to be compatible is

$$[f, g] = \frac{\delta(f, g)}{\delta(x, p)} + p \frac{\delta(f, g)}{\delta(z, p)} + \frac{\delta(f, g)}{\delta(f, q)} + q \frac{\delta(f, g)}{\delta(z, q)} = 0$$

[10 mks]

QUESTION FOUR

[20 Marks]

- (a) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through
- (i) the straight line $x = 1, y = z$. [7 mks]
 - (ii) the parabola $y^2 = z, x = 0$ [7 mks]
- (b) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. [6mks]

QUESTION FIVE

[20 Marks]

- (a) Find the complete integral of $(p^2 + q^2)y = qz$ using Charpit's method. [6 mks]
- (b) Apply Jacobi's method to find the complete integral of

$$x^2 p_1^2 p_2^2 p_3^2 + p_1^2 p_2^2 - p_3^2 = 0.$$

[7 mks]

- (c) Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the parabola $2x = y^2 + z^2$ is the cylinder which touches it along its section by the plane $y + 1 = 0$. [7 mks]

END
ALL THE BEST

MMA 405: PDE

Course Instructor: MOSE BAGASA

Course Description

- Simultaneous differential equations of first order in 3 variables
- Pfaffian differential equations
- Linear first order pde
- Non-linear first order pde
- Compatible systems of first order pde