TOM MBOYA UNIVERSITY

UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH I.T

MAC 203 :FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

Instructions:

- Answer QUESTION ONE (compulsory) and any other TWO.
- Proofs should be written carefully.
- Cheating is NOT allowed and will be harshly punished.
- Observe further instructions on the answer booklet.

QUESTION ONE [30 Marks]

(a) The mortality of a certain population is governed by the life table function $l_x = 100 - x$, $0 \leq x \leq 100$. Calculate the values of the following expressions

(i) ${}_{10}p_{35}$

(ii) μ_{35}

(iii) $P(T_{35} < 35)$

(iv) $P(K_{35} < 35)$

[4 marks]

(b) Describe the main features of an endowment assurance contract. [3 marks]

(c) A life insurance company issues a 3-year term assurance contract to a life aged exactly 42. The sum assured of 10,000 is payable at the end of the policy year of death. Calculate the expected present value of these benefits assuming AM92 Select mortality and an interest rate of 6% pa effective. [4 marks]

(d) An impaired life aged 40 experiences 5 times the force of mortality of a life of the same age subject to standard mortality. A two-year term assurance policy is sold to this impaired life, and another two-year term assurance is sold to a standard life aged 40. Both policies have a sum assured of £10,500 payable at the end of the year of death. Calculate the expected present value of the benefits payable to each life assuming that standard mortality is AM92 Ultimate and interest is 4% pa. [4 marks]

(e) A level annuity of £1,500 pa is to be paid continuously to a 40-year-old male for the rest of his life. On the basis of 4% pa interest and AM92 Ultimate mortality, calculate the expected present value of this annuity. [4 marks]

(f) You are given the following survival distribution:

$$q_x = 0.15, \quad q_{x+1} = 0.5, \quad q_{x+2} = 1$$

Find A_x if $i = 0.15$. [3 marks]

(g) You are given $q_{50} = 0.00529$ and $i = 0.055$. Find the variance of the present value random variable for a one-year term insurance issued to (50) with face amount 1000 USD. [4 marks]

(h) A 50-year-old woman purchases a deferred annuity to provide herself with an income of £15,000 pa , paid annually in advance from age 70 until death. Calculate the expected present value of the benefits from this deferred annuity, using PFA92C20 mortality, and an interest rate of 4% pa effective. [4 marks]

QUESTION TWO [20 Marks]

(a) An annuity is payable continuously throughout the lifetime of a person now aged exactly 60, but for at most 10 years. The rate of payment at all times t during the first 5 years is £10,000 pa, and thereafter it is £12,500 pa. The force of mortality of this life is 0.035 pa between the ages of 60 and 65, and 0.046 pa between the ages of 65 and 70. Calculate the expected present value of this annuity assuming a force of interest of 0.066 pa. [5 marks]

(b) Write down a single term expression to represent the present value of a deferred annuity-due of 1 per annum payable to a life now aged x, with a deferment period of n years. Hence derive a formula for the variance of the present value of this contract. [10 marks]

(c) A life office has just sold a 25-year term assurance policy to a life aged 40. The sum assured is £60,500 and is payable at the end of the year of death. Calculate the variance of the present value of this benefit, assuming AM92 Ultimate mortality and 4% pa interest. [5 marks]

QUESTION THREE [20 Marks]

(a) For a 20-year pure endowment of 1 on (x), you are given:

(i) A is the present value random variable at issue of the benefit payment.

(ii) $Var[Z] = 0.055E[Z]$

(iii) ${}_{20}p_x = 0.655$

Calculate i. [6 marks]

(b) Calculate the expectation and standard deviation of the present value of the benefits from each of the following contracts issued to a life aged exactly 40, assuming that the annual effective interest rate is 4% and AM92 Ultimate mortality applies:

(i) a 20-year pure endowment, with a benefit of £16,500

(ii) a deferred whole life assurance with a deferred period of 20 years, under which the death benefit of £20,020 is paid at the end of the year of death, as long as this occurs after the deferred period has elapsed.

[7 marks]

(c) You are given:

(i) $A_x = 0.28$

(ii) $A_{x+20} = 0.46$

(iii) $A_{x:\overline{20}|}$

(iv) $i = 0.036$

Assume deaths are uniformly distributed over each year of age. Calculate $1000\bar{A}_{x:\overline{20}|}$. [7 marks]

QUESTION FOUR [20 Marks]

(a) An endowment assurance contract with a term of 10 years pays a sum assured of £100,000 immediately on death and a sum of £50,000 on survival for 10 years. Calculate the expected present value and variance of this contract.
Basis:
Mortality: $\mu_x = 0.035$ throughout [8 marks]

(b) Z is the present value random variable for a special whole life insurance with death benefits payable at the moment of death of (x) .You are given for $t \geq 0$:

(i) $b_t = e^{0.055t}$

(ii) $\delta_t = 0.056$

(iii) $\mu_x(t) = 0.012$

Calculate $Var[Z]$. [6 marks]

(c) Show that if Z_1 and Z_2 are teh present value random variables for n-year term insurance and n-year pure endowment, the $Z_1 \times Z_2 = 0$. [6 marks]

QUESTION FIVE [20 Marks]

(a) You are given:

(i) X is the present value random variable for a 25-year term insurance of 7 on (35) .

(ii) Y is the present value random variable for a 25-year deferred, 10-year term insurance of 4 on the same life.

(iii) $E[X] = 2.83, \quad E[Y] = 0.124$

(iv) $Var[X] = 5.764, \qquad Var[Y] = 0.104$

Calculate $Var[X + Y]$. [5 marks]

(b) (i) Let Z be a random variable representing the present value of the benefits payable under an immediate life annuity that pays 1 per year in advance, issued to a life aged x . Show that

$$Var(Z) = \frac{1}{d^2}\left({}^2A_x - (A_x)^2\right)$$

where ${}^2A_x$ is an assurance calculated at rate of interest which you should specify.

(ii) A life office issues such a policy to a life aged exactly 65. The benefit is £275 per annum. Calculate the standard deviation of the annuity.

[7 marks]

(c) Z_1 is the present value random variable for an $n-$year term insurance of 1 on the life of (x). Z_2 is the present value random variable for an $n-$year endowment insurance of 1 on the life of (x). You are given

(i) $v^n = 0.22$

(ii) ${}_np_x = 0.56$

(iii) $E[Z_1] = 0.234$

(iv) $Var[Z_1] = 0.081$

(v) Death benefits are payable at the moment of death.

Calculate $Var[Z_2]$ [8 marks]