TOM MBOYA UNIVERSITY

KNOWLEDGE FOR SUSTAINABLE INNOVATION ENTERPRISE

FACULTY OF BIOLOGICAL AND PHYSICAL SCIENCES

2024/2025 ACADEMIC YEAR UNIVERSITY EXAMINATIONS

FIRST YEAR FIRST SEMESTER UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE) WITH IT

SPH 101: MECHANICS

DATE: 20/12/2024 **TIME: 3 HOURS**

INSTRUCTIONS

- Answer question one (**compulsory**) and any other two (2) questions.
- Do not write on this question paper.
- Cheating is a serious offence.

CONSTANTS

Gravitational constant (G) = $6.673 \times 10^{-11}\ Nm^2kg^{-2}$
Mass of Earth (M_E) = $5.98 \times 10^{24}kg$
Radius of Earth (R_E) = $6.37 \times 10^{6}m$
Free-fall acceleration (g) = $9.81ms^{-2}$

EXAMINATION CHEATING IS A CRIME

QUESTIONS ONE (30 MARKS)

a) (i) What is the magnitude of the centripetal acceleration of an object on Earth's equator due to the rotation of Earth?

(3 marks)

(ii) What would Earth's rotation period have to be for objects on the equator to have a centripetal acceleration of magnitude $9.8ms^{-2}$?

(2 marks)

b) Show that in motion of a particle at constant acceleration, the displacement

$$\vec{x} - \vec{x}_0 = \vec{v}t - \frac{1}{2}\vec{a}t^2.$$

(3 marks)

c) To keep an object moving along a uniform curved path at a constant speed v, requires a force F, which varies directly as its mass m, its speed v and radius r of the curved path. By applying dimensional analysis, determine the required expression for F.

(3 marks)

d) A $12N$ force with a fixed orientation does work on a particle as the particle moves through three-dimensional displacement $\vec{d} = 2m\ \hat{i} - 4m\ \hat{j} + 3m\ \hat{k}$. What is the angle between the force and the displacement if the change in the particle's kinetic energy is $-30J$?

(4 marks)

e) The acceleration of a particle moving only on a horizontal xy plane is given by $\vec{a} = (3t\ \hat{i} + 4t\ \hat{j})ms^{-2}$. At $t = 0$, the position vector $\vec{r} = 20m\ \hat{i} + 40m\ \hat{j}$ locates the particle, which then has a velocity $\vec{v} = 5ms^{-1}\hat{i} + 2ms^{-1}\hat{j}$. At $t = 4s$, what is its position vector in unit-vector notation?

(4 marks)

f) In unit-vector notation, what is the torque about the origin on a plane located at $(-2m, , 0m, 4m)$ if the torque is due to a force $\vec{F}$ whose only component is $F_x = 6N$.

(3 `marks`)

g) Left over from the big-bang beginning of the universe, tiny black holes might still wander through the universe. If one with mass of 1×10^{11}kg and radius of only 1×10^{-16}m reached the Earth, at what distance from your head would its gravitational pull on you match that of Earth?

(4 `marks`)

h) The system in Figure 1 is in equilibrium, with the string in the centre exactly horizontal.

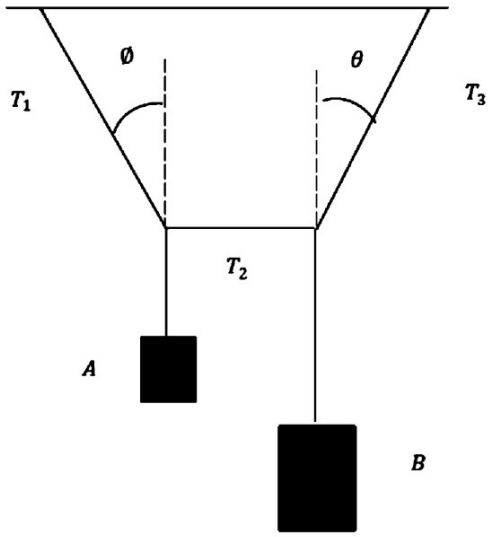


Figure 1:

Block A weighs $40N$, B weighs $50N$ and angle $\phi = 35^0$. Find the tension on each cord and angle θ.

(4 `marks`)

QUESTION TWO (20 MARKS)

a) Figure 2 shows a double incline plane supporting blocks of masses $m_1 = 100kg$ and $m_2 = 50kg$.

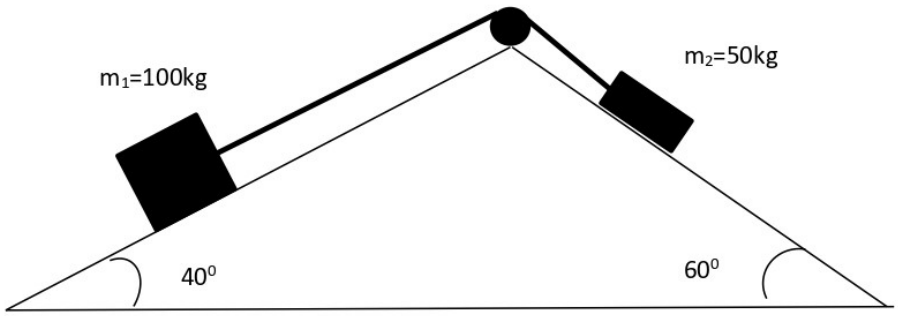


Figure 2:

The system is released from rest in the position shown in Figure 2. If the coefficient of kinetic friction between m_1 and the surface it rests on is 0.3, while the the surface on which m_2 rests is smooth, determine;

(i) the acceleration of the system and

(8 marks)

(ii) the tension on the cord joining the masses,

(3 marks)

when the resultant force is acting along the $+y$ direction.

b) (i) A projectile is in motion in the xy plane. Determine the parabolic equation of the trajectory of the projectile, expressed in terms

of angle of projection, θ_0, free-fall acceleration, g, and speed of projection, v_0.

(4 marks)

(ii) Applying the result in question 2 (b) (i), find the expression for maximum vertical displacement of the projectile.

(5 marks)

QUESTION THREE (20 MARKS)

a) Figure 3 shows a pendulum of length $L = 1.25m$. Its bob has a speed v_0 when the cord makes an angle $\beta_0 = 40^0$ with the vertical. By applying conservation of mechanical energy, find;

(i) the speed of the bob when it is at its lowest position if $v_0 = 8ms^{-1}$ and

(6 marks)

(ii) the least value that v_0 can have if the pendulum is to swing down and then up to a horizontal position.

(4 marks)

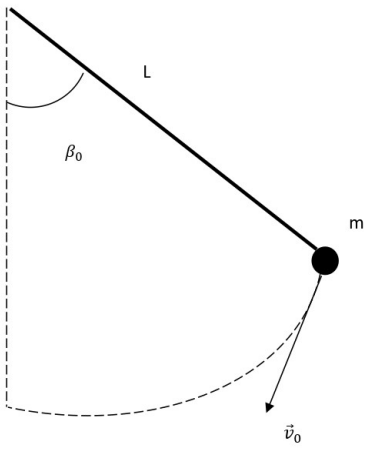


Figure 3:

b) A satellite, moving in an elliptical orbit, is $600km$ above the Earth's surface at its furthest point and $250km$ above its closest point. Calculate;

(i) the semi-major axis and

(5 marks)

(ii) the eccentricity of the orbit.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Figure 4 shows a wooden beam leaning out against a wall that has negligible friction. Distance a is $0.914m$ and distance L is $2.10m$.

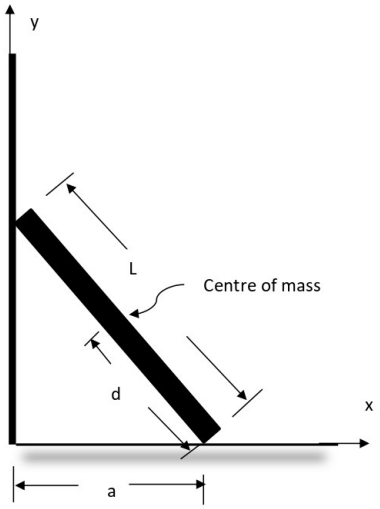


Figure 4:

Its centre of mass is at a distance $d = 0.94m$ from the horizontal floor contact point. If the wooden beam is on the verge of sliding, what is

the coefficient of static friction between the base of the wooden beam and the floor?

(10 marks)

b) A ball is shot from the ground into air. At a height of $9.1m$, its velocity is $\vec{v} = 7.6ms^{-1}\ \hat{i} + 6.1ms^{-1}\ \hat{j}$.

(i) What is the the magnitude and angle below the horizontal of the ball's velocity just before it hits the ground?

(5 marks)

(ii) What total horizontal distance does the ball travel?

(5 marks)

QUESTION FIVE (20 MARKS)

a) A spacecraft passed $120AU$ mark in its distance from the Sun. At this time, a Star intercepts the spacecraft. The Star enters its course and starts to orbit the Sun from that point and sets the heading such that it will reach the perihelion at the Earth's location. What is the length of time in years that it will take to get to Earth?

(9 marks)

b) (i) A particle moves along a circular path over a horizontal xy co-ordinate system, at constant speed. At a time $t_1 = 4s$, it is at a point $(5m, 6m)$ with velocity $3ms^{-1}\ \hat{j}$ and acceleration in the $+x$ direction. At time $t_2 = 10s$, it has velocity $-3ms^{-1}\ \hat{i}$ and acceleration in the $+y$ direction. Find the x, y coordinates of the centre of the path.

(6 marks)

(ii) Two $2kg$ bodies, A and B, collide. The velocities before collision are $\vec{v}_A = 15ms^{-1}\ \hat{i} + 30ms^{-1}\ \hat{j}$ and $\vec{v}_B = -10ms^{-1}\ \hat{i} + 5ms^{-1}\ \hat{j}$. After collision $\vec{v}_A{}' = -5ms^{-1}\ \hat{i} + 20ms^{-1}\ \hat{j}$. What is the change in total kinetic energy of the system?

(5 marks)