TOM MBOYA UNIVERSITY
MAC 415: SURVIVAL MODELS
REGULAR EXAMINATION

QUESTION ONE (30 MARKS)

(a) You are given the following excerpt of a life table:

x	l_x	d_x
20	96178.01	99.0569
21	96078.95	102.0149
22	95976.93	105.2582
23	95871.68	108.8135
24	95762.86	112.7102
25	95650.15	116.9802

Calculate the following: [5 Marks]

(i) ${}_5p_{20}$

(ii) q_{24}

(iii) ${}_{4|1}q_{20}$

(b) The probability density function for the future lifetime of a life age 0 is given by;

$$f_0(x) = \frac{\alpha\lambda^{\alpha}}{(\lambda+x)^{\alpha+1}}, \qquad \alpha,\lambda > 0$$

(i) Show that the survival function for a life age 0, $S_0(x)$ is $\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}$. [3 Marks]

(ii) Derive an expression for μ_x. [2 Marks]

(iii) Derive an expression for $S_x(t)$. [2 Marks]

(iv) Using (b) and (c), or otherwise, find an expression for $f_x(t)$. [3 Marks]

(c) Mortality of a group of lives is assumed to follow Gompertz law. Calculate, μ_x for a 30 year old and a 70 year old, given that μ_x is 0.003 for a 50-year old and 0.01 for a 60-year old. [5 Marks]

(d) Examine the column of dx shown in the English Life Table No. 15 (Males) in the Formulae and Tables for Examinations (Pages 68-69);
Describe the key characteristics of this mortality table using the data to illustrate your points. [5 Marks]

(e) Data are censored if we do not know the exact values of each observation but we do have information about the value of each observation in relation to one or more bounds. Describe the following types of censoring, giving examples; [5 Marks]

(i) Left censoring

(ii) Interval censoring

(iii) Right censoring

QUESTION TWO (20 MARKS)

(a) Write down integral equations for the mean and variance of the complete future lifetime at age x, T_x. [5 Marks]

(b) Explain what is meant by a proportional hazards model. [4 Marks]

(c) Outline three reasons why the Cox proportional hazards model is widely used in empirical work. [4 Marks]

(d) The Cox proportional hazards model is to be used to model the rate at which students leave a certain profession before qualification. Assuming they stay in the profession, students will qualify three years after joining the profession. In the fitted model, the hazard depends on the time, t, since joining the profession and three covariates. The covariates, their categories and the fitted parameters for each category are shown in the table below:

Covariate	Possibility	Parameter
Size of employer	Large	0
	Small	0.4
Degree studied	None	0.3
	Science	–0.1
	Arts	0.2
	Other	0
Location	London	0
	Other UK	–0.3
	Overseas	0.4

(a) Defining clearly all the terms you use, write down an expression for the hazard function in this model. [4 Marks]

(b) State the class of students that is most likely to proceed to qualification under this model, and that which is least likely. [3 Marks]

QUESTION THREE (20 MARKS)

(a) Turn to ELT15 (Males) in your Tables and find the complete expectation of life e^0 for the following male lives: [6 Marks]

(i) a new-born baby

(ii) a 21-year old actuarial student

(iii) a 70-year old pensioner

(b) A certain profession admits new members to the status of student. Students may qualify as fellows of the profession by virtue of passing a series of examinations. Normally student members sit the examinations whilst working for an employer. There are two sessions of the examinations each year. An employer provides study support to student members of the profession. It wishes to assess the cost of providing this study support

and therefore wishes to know the average time it can expect to take for its students to qualify. The employer has maintained records for 23 of its students who all sat their first examination in the first session of 2003. The students progress has been recorded up to and including the last session of 2009. The following data records the number of sessions which had been held before the specified event occurred for a student in this cohort:
Qualified : 6, 8, 8, 9, 9, 9, 11, 11, 13, 13, 13
Stopped studying : 4, 5, 8, 11, 14
The remaining seven students were still studying for the examinations at the end of 2009.

(a) Determine the median number of sessions taken to qualify for those students who qualified during the period of observation. [2 Marks]

(b) Calculate the Kaplan-Meier estimate of the survival function, S(t), for the hazard of qualifying, where t is the number of sessions of examinations since 1 January 2003. [10 Marks]

(c) Hence estimate the median number of sessions to qualify for the students of this employer. [2 Marks]

QUESTION FOUR (20 MARKS)

(a) You are given $\mu_x = 0.01$ for all $x \geq 0$. Calculate the following;

(i) e_x^0. [4 Marks]

(ii) $Var[T_x]$. [6 Marks]

(b) A new weedkiller was tested which was designed to kill weeds growing in grass. The weedkiller was administered via a single application to 20 test areas of grass. Within hours of applying the weedkiller, the leaves of all the weeds went black and died, but after a time some of the weeds re-grew as the weedkiller did not always kill the roots.
The test lasted for 12 months, but after six months five of the test areas were accidentally ploughed up and so the trial on these areas had to be discontinued. None of these five areas had shown any weed re-growth at the time they were ploughed up.

- Ten of the remaining 15 areas experienced a re-growth of weeds at the following durations (in months): 1, 2, 2, 2, 5, 5, 8, 8, 8, 8.
- Five areas still had no weed re-growth when the trial ended after 12 months.

(i) Describe, giving reasons, the types of censoring present in the data. [4 Marks]

(ii) Estimate the probability that there is no re-growth of weeds nine months after application of the weedkiller using the Nelson-Aalen estimator. [6 Marks]

QUESTION FIVE (20 MARKS)

(a) If $\mu_x = 0.01908(x-70)$ for $x \geqslant 55$, Calculate ${}_5q_{60}$ [7 Marks]

(b) Calculate the exact values of the complete and curtate expectation of life for a newborn animal subject to a constant force of mortality of 0.05 per annum. [6 Marks]

(c) Express q_{30}, e_{30} and $-5p_{35}$ in terms of probabilities of the random variable K_{30} which represents the curtate future lifetime of a life aged exactly 30. [7 Marks]