TOM MBOYA UNIVERSITY
UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR,

FOURTH YEAR S FIRST SEMESTER EXAMINATION FOR THE DEGREE OF B.SC IN APPLIED STATISTIC, ACTUARIAL SCIENCE, MATHEMATICAL ECONOMICS
COURSE CODE: MIT 401
COURSE TITLE: BAYESIAN MODELLING
(MAIN EXAM, 2023)

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INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and **ANY THREE** from Questions in section B.

All questions in section B carry Equal Marks

Duration of the examination: 3 hours

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SECTION A (25 marks):

QUESTION ONE 30 mks

a) Explain what the following aspect represent in BUGs language and illustrate using example?
   i. $<-$ (2mks)
   ii. ~ (2mks)
b) Define the following terms
   i. Likelihood probability distribution (2 mks)
   ii. Prior probability distribution (2 mks)
   iii. Posterior probability distribution (2 mks)
c) Distinguish between the concept prior elicitation and conjugate prior as used in Bayesian modelling (4mks)
d) State the roles of the following R console functions in relation to the Bayesian modelling
   i. Xdesign() (1mk)
   ii. Bayes.lin.reg() (1mk)
   iii. nvaricp() (1mk)
e) Outline the six steps of compiling the model and simulating values in a Bayesian model using winBUGs software (3mks)
f) In winBUGs terminology, explain the concept "setting the monitored parameters" (2mks)
g) Suppose we want to know the probability of getting 8 or more heads when we toss a fair coin 10 times using **monte carlo approach.**
   i. Calculate the true distribution using the algebraic approach pr($x \geq 8$) (4mks)
   ii. How can the above model be represented in the BUGs language (4mks)

SECTION B. ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MKS)

a) Let X be a binomial distribution (n ,p) that is

F (x/p) = $\binom{n}{x} p^x (1-p)^{n-x}$ x = 0, 1, 2, 3…n

Let the prior distribution π (p) be a beta distribution (α, β)

Π (p) = $\frac{\Gamma a+\beta}{\Gamma a+\beta} p^{a-1}(1-p)^{\beta-1}$ 0 < p < 1

Obtain the;

i. Posterior distribution π(p/x) (8 mks)

ii. Posterior expectation E(p/x) (7 mks)

iii. Write the BUGs syntax to output the above model? (5 mks)

QUESTION THREE (20 MKS)

a) Distinguish between the Bayesian framework and the frequentist framework? (4 mks)

b) A new HIV test is claimed to have 95% sensitivity and 98% specificity, in a population with an HIV prevalence of 1/1000, what is the chance that patient testing positive actually has HIV? (5 mks)

c) Given prior distribution for proportion of responders in one year θ was Beta [9.2,13.8]. consider a situation before giving 20 patients the treatment. What is the chance if getting 15 or more responders?

i. Write a BUGs syntax to output the above model? (6 mks)

ii. Draw a graphical representation of the model above? (5 mks)

QUESTION FOUR (20 MKS)

Let y_i ~bin(n_i, π_i) where the probability π_i have a beta distribution i.e $\pi_i \sim \beta(\alpha_i, \beta_i)$. Therefore $\pi_i \sim \beta(\alpha_i, \beta_i)$=$\frac{1}{\beta(\alpha_i,\beta_i)} \pi^{\alpha-1}(1-\pi)^{\beta-1}$. If x~ $\beta(\alpha_i, \beta_i)$ then E(x)=$\frac{\alpha}{\alpha+\beta}$ and the variance var(x)=$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

Show that;

i. E(π_i)=θ (6 mks)

ii. Var(π_i)=E(π_i^2)-$(E\pi_i)^2$ (7 mks)

iii. E(y_i)=$n_i\theta$ (3 mks)

iv. Write the BUGs language to output the model above (4 mks)

QUESTION FIVE (20 MKS)

a) What do you understand by the term Bayesian modelling? (2 mks)
b) List any three limitations or drawbacks to using winBUGs in Bayesian modelling?(3mks)
c) Regional water companies in the UK are required to take routine measurements of trihalomethane (THM) concentrations in tap water samples for regulatory purposes. Samples are tested throughout the year in each water supply zone. Suppose we want to estimate the average THM concentration in a particular water zone, Z. Two independent measurements x_{z1} and x_{z2} is 130$\mu g/l$. Suppose we know that the assay measurement error has a standard deviation $\delta = 5\mu g/l$.
   What should we estimate the mean THM concentration to be in this water Zone? (8 mks)
d) Data below are the number of flying bomb hits on London during World War II in 36 km^2 area of south london. The area was partioned into 0.25 km^2 grid square and number of bombs falling in each grid was counted.

Hits, x	0	1	2	3	4	7
No of areas	229	211	93	35	7	1

   Total hits =537,
   Use the Poisson modelling with a constant hit rate to obtain the posterior distribution for the dataset above. (7 mks)