

TOM MBOYA UNIVERSITY
UNIVERSITY EXAMINATION

SEMESTER 1, 2023/2024

UNIT CODE: MMA 102 UNIT TITLE: CALCULUS I

TIME: 3HOURS

INSTRUCTION: ANSWER QUESTION ONE (COMPULSORY AND ANY OTHER TWO)

QUESTION ONE (30MKS)

a) Differentiate using power rule $y = (9 - 6x + x^2)(x^3 - x + 1)$ (4mks)

b) Evaluate the limits of the following:

i. $\lim_{x \rightarrow 2} \frac{4-x^2}{2-x}$ (3mks)

ii. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$ (4mks)

c) i) Let f be a function. Define f (2mks)

ii) Confirm whether $f(x) = x^2$ is a function (4mks)

d) Find the range of the following functions:

i. $g(x) = \frac{1}{x+2}$, $x \neq -2$ (3mks)

ii. $h(x) = \frac{1}{x-4}$, $x \neq 4$ (3mks)

e) Given that $f: x \mapsto 4 - x$, find :

i. domain of f (2mks)

ii. range of f (2mks)

f) Using quotient rule, differentiate $y = x^{-2}$ (3mks)

QUESTION TWO (20MKS)

a) Using first principle, find the derivative of the following:

i. $y = 2x + 1$ (3mks)

ii. $g(x) = x^3$ (3mks)

iii. $f(x) = 3x^2 - x + 1$ (5mks)

b) The area of a circle is related to its radius by the equation $A = \pi r^2$. Find the instantaneous rate of change in area with respect to radius when the radius is 3cm (5mks)

c) With the concept of implicit differentiation, find $\frac{dy}{dx}$ given $xy = y + 5$ (4mks)

QUESTION THREE(20MKS)

a) Find $\frac{dy}{dx}$ using the product rule in the following cases:

i. $y = (x^3 - 1)(x^3 + 3)$ (4mks)

ii. $y = x(x - 1)^{\frac{1}{2}}$ (4mks)

iii. $xy^2 = y - 5$ (4mks)

b) With the help of a sketch, compute the domain of the inverse of $p(x) = \frac{1}{x-1}, x \neq 1$ (4mks)

c) Evaluate $\lim_{x \rightarrow 1} \frac{x-1}{(\sqrt{x+3})-2}$ (4mks)

QUESTION FOUR (20MKS)

a) With your knowledge of chain rule, find the derivatives of the following:

i. $y = (x^2 - 1)^3$ (4mks)

ii. $y = (1 - x)^{\frac{2}{3}}$ (4mks)

b) The functions f, g, h are defined as $f(x) = 7 - 2x$, $g(x) = 3x - 1$, $h(x) = \frac{1}{x}; x \neq 0$. Compute in terms of x the composite functions :

i. $gfh(x)$ (3mks)

ii. $hhgg(x)$ (3mks)

iii. $gff(x)$ (3mks)

QUESTION FIVE(20MKS)

a) A rectangular sheet of thick cardboard is 80cm by 50cm. A square of side x cm is cut away from each corner of the sheet which is then folded to form a rectangular box of volume V .

i. Show that $V = 4000x - 260x^2 + 4x^3$ (4mks)

ii. Compute the greatest volume possible of the box when it occurs (the value of x for which the volume is greatest) (6mks)

b) A spherical balloon is filled with air at the rate of $8\text{cm}^3/\text{s}$. At what rate is its radius increasing when

i. $r = 12\text{cm}$ (4mks)

ii. $V = 1500\text{cm}^3$ (4mks)

c) Find $\frac{dy}{dx}$ given that $y = e^{2x}$ where e is an exponential function (2mks)