TOM MBOYA UNIVERSITY

UNIVERSITY EXAMINATION 2024 / 2025

MAIN EXAMINATIONS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE & BACHELOR OF EDUCATION

MMA 201: LINEAR ALGEBRA II

Instructions:

- Answer QUESTION ONE (30 marks) and any other TWO questions (20 marks each).
- Proofs should be written carefully.
- Cheating is NOT allowed and will be harshly punished.
- Observe further instructions on the answer booklet.

QUESTION ONE (Compulsory) [30 Marks]

a.) (i) Define a vector space. (2 marks)

(ii) State any three properties of a vector space. (3 marks)

b.) Show that the n-space ($\mathbb{R}^n$) with the inner product defined as $\langle u, v\rangle = u_1v_1+u_2v_2+...+u_nv_n$ where $u = (u_1, u_2, ..., u_n)$ and $v = v_1, v_2, ..., v_n$ is an inner product space. (4 marks)

c.) (i) Define an orthonormal set. (2 marks)

(ii) Verify that the set $S = \{(0,1,0),(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}})\}$ is orthonormal in $\mathbb{R}^3$ (3 marks)

(d.) Let $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$. Show that A is an orthogonal matrix. (5 marks)

e.) Let $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$, Show that $A^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$ is a linear combination of A and I (identity matrix). (4 marks)

f.) (i) Distinguish between a linear transformation and isomorphism . (3 marks)

(ii) Show that the transformation $T(A) = S^{-1}AS$ from $\mathbb{R}^{2x2}$ to $\mathbb{R}^{2x2}$ is an isomorphism where $S = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. (4 mark)

QUESTION TWO (20 MARKS)

a.) Verify that the following is an inner product on polynomial P_2

$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)dx$$

where $p(x)q(x) \in P_2$ (8 marks)

b.) Use the G-S process to covert $S = \{(1,1,1),(-1,1,0),(1,2,1)\}$ into orthonormal basis in $\mathbb{R}^3$ (12 marks)

QUESTION THREE (20 MARKS)

a.) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (x_1+x_2, -x_1, 0)$. Given $A = \{(1,3),(-2,4)\}$ and $A' = \{(1,1,1),(2,2,0),(3,0,0)\}$;

(i) Find the matrix of T with respect to A and A' (8 marks)

(ii) Use the matrix obtained in (i)above to find the transformation $T(8,3)$ (6 marks)

b.) Verify that Cayley Hamilton theorem holds in $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ (6 marks)

QUESTION FOUR(20 MARKS)

a.) Show that the matrix M that commute with $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$ form a subspace of $\mathbb{R}^{2x2}$. (8 marks)

b.) Find the eigen values, eigen vectors and the basis of eigen space of $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 0 & 0 & 2 \end{pmatrix}$ (12 marks)

QUESTION FIVE (20 MARKS)

Diagonalize the matrix $M = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$