



**TOM MBOYA UNIVERSITY
UNIVERSITY EXAMINATION
2022/2023 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE
OF BACHELOR OF EDUCATION (SCIENCE)
SPH 313:CLASSICAL MECHANICS**

DURATION OF THE EXAMINATION: 3 HOURS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE** (COMPULSORY) AND ANY OTHER **TWO** QUESTIONS

CONSTANTS

Proton mass (m_p) = 1.67×10^{-27} kg

Speed of light in a vacuum (c) = 3.0×10^8 m/s

QUESTION ONE (30 MARKS)

- a) Prove that the force $\vec{F} = r^2\vec{r}$ is conservative and find the corresponding scalar potential.
(3 mark)
- b) A particle of mass m moves according to the equation $x = a \cos(\omega t)$, $y = a \sin(\omega t)$, $z = c$ where a, c, ω are constants. Determine its angular momentum in terms of m, ω, x, y and z .
(3 mark)
- c) Show that the angular momentum $\vec{J}$ of a system of particles can be expressed in the form $\vec{J} = \vec{J}_{com} + \vec{R} \times \vec{P}$
where
 $\vec{J}_{com}$ is the angular momentum about the center of mass
 $\vec{R}$ is the position vector of the center of mass
 $\vec{P}$ is the total linear momentum
(3 marks)
- d) A Muon is an elementary particle that is unstable and decays rather quickly with a mean life-time of $2.2\mu s$. If the Muon is accelerated to a speed of 20% the speed of light in a vacuum, find the life-time of Muon at this speed.

(3 marks)

- e) Show that the total energy E of a particle of mass m acted upon by a central force takes the form

$$E = \frac{J^2}{2m} \left[u^2 + \left(\frac{du}{d\theta} \right)^2 \right] + V(r)$$

where $V(r)$ is the potential energy, J the angular momentum and (r, θ) the polar coordinates of the particle; $u = \frac{1}{r}$.

(3 marks)

- f) Determine the constraint equations in the case of a double pendulum and specify its generalised coordinate.

(3 mark)

- g) If $y(x)$ is an extremal of a functional I and that it defines the path between two points on a flat surface, determine $F(x, y, \frac{dy}{dx})$ that satisfies the Euler-Lagrange equation.

(3 marks)

- h) Write down the Lagrange equation if the Lagrangian has the form

$$L = -1 \left(1 - \dot{q}^2 \right)^{\frac{1}{2}}$$

(3 marks)

- i) The Lagrangian of a particle of mass m moving in a force field of potential $V(r, \theta, \phi)$ is

$$L = \frac{1}{2}m \left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2(\theta)\dot{\phi}^2 \right) - V(r, \theta, \phi) .$$

Determine the Hamilton's equation in spherical polar coordinates (r, θ, ϕ) .

(3 mark)

- j) A Proton is accelerated to a velocity of $0.85c$. Determine the relativistic kinetic energy of the particle.

(3 marks)

QUESTION TWO (20 MARKS)

- i) Two particles of mass m_1 and m_2 are located on a frictionless double incline and connected by an inextensible massless string passing over a smooth peg as shown in Fig.1.

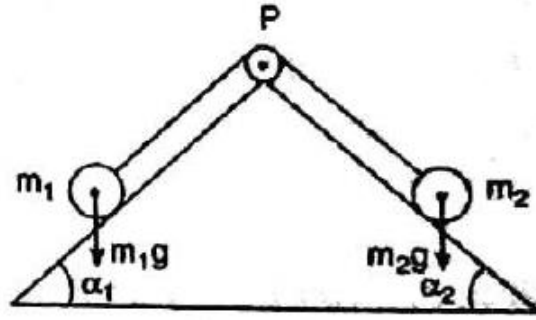


Figure 1:

Use the principle of virtual work to show that for equilibrium, we must have

$$\frac{\sin(\alpha_1)}{\sin(\alpha_2)} = \frac{m_2}{m_1}$$

(10 marks)

- ii) Apply the variational principle to show that the shortest path between points $A_1(x_1, y_1)$ and $B(x_2, y_2)$ is of the form

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1) + y_1$$

(10 marks)

QUESTION THREE (20 MARKS)

- i) Consider a hoop rolling down an incline plane without slipping.

- a) Determine the generalised coordinates.

(2 marks)

- b) What is the Lagrangian?

(3 marks)

- c) Find its acceleration and frictional force of constraint by using the method of undetermined multipliers.

(5 marks)

- ii) Show that the path difference ΔN according to Michelson-Morley experiment is

$$\Delta N = \frac{Lv^2}{c^2}$$

where L is the pathlength, v the velocity of Earth around the Sun and c the speed of light.

(10 marks)

QUESTION FOUR (20 MARKS)

- i) Obtain the equation of motion of a simple pendulum by using the Lagrangian method and hence deduce the formula for its time period for small amplitude oscillations.

(10 marks)

- ii) Use Hamilton's equation to obtain the motion of a particle of mass m down a frictionless inclined plane of angle α .

(10 marks)