# TOM MBOYA UNIVERSITY

## UNIVERSITY EXAMINATIONS
## 2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH I.T

## MAC 303 : ACTUARIAL MATHEMATICS II

**Instructions**:

- Answer QUESTION ONE (compulsory) and any other TWO.
- Proofs should be written carefully.
- Cheating is NOT allowed and will be harshly punished.
- Observe further instructions on the answer booklet.

# QUESTION ONE [30 Marks]

(a) A whole life insurance policy of face amount 1,000 USD is issued to (30) . On the basis of some mortality table and a 6% interest rate, the following values are determined relative to the first year of the policy : (i) benefit premium = 20 UDS (due at the start of the year) , and (ii) terminal benefit reserve at the end of the year = 18 USD . What is $q_{30}$? [4 marks]

(b) Explain what it means for the last survivor status $\overline{55:65}$ to remain active for at least 10 years. [3 marks]

(c) You are given : (i) ${}_5p_{50} = 0.7$, (ii) ${}_5p_{60} = 0.6$ (iii) $q_{55} = 0.03$, (iv) $q_{65} = 0.04$ (iv) $T(50)$ and $T(60)$ are independent. Calculate ${}_5|q_{\overline{50:60}}$ [4 marks]

(d) A man aged 60 exact and a woman aged 65 exact wish to purchase an annuity that provides:

- £25,000 pa payable while they are both alive,
- £20,000 pa payable for the remainder of the woman?s life, if the man dies first,
- £15,000 pa payable for the remainder of the man?s life, if the woman dies first.

All the annuity payments are made annually in arrears.

(i) Write down an expression for the present value random variable of this benefit. [2 marks]

(ii) Calculate the expected present value of this annuity benefit, using the following **basis**: [2 marks]
Mortality: PMA92C20 for the male life, PFA92C20 for the female life

Interest: 4% pa effective

(e) (40) and (50) are independent lives whose survival is based on the AM92 Tables. Find the probability that the first death occurs between 10 and 20 yeas from now. [4 marks]

(f) A life insurance company provides the following benefits:

- an annuity, on survival to age 65, of £15,000 per annum payable monthly in advance
- a spouse?s annuity of £8,000 per annum payable monthly in advance on the death of the policyholder, provided that the policyholder survives to age 65.

No benefit is payable if the policyholder dies before age 65.

Calculate the single premium in respect of a female policyholder currently aged exactly 50 who has a male spouse currently aged exactly 53. **Basis:** Mortality : PFA92C20/PMA92C20

Interest: 4% p.a. Ignore expenses. [6 marks]

(g) A common Shock model has $\lambda = 0.03$, and $T^*(x)$ and $T^*(y)$ are both subject to a constant force of mortality of .01 . Find

(i) the probability that $(x)$ and $(y)$ will die within 10 years, and

(ii) the probability that $(x)$ and $(y)$ will die within 10 years as a result of the common shock.

[5 marks]

# QUESTION TWO [20 Marks]

(a) For two independent lives $(x)$ and $(y)$, you are given: (i) Mortality follows DeMoivre's law (ii) $(x)$ and $(y)$ were both born on January 1, (iii) $0 < x < y < \omega$. Formulate an expression for the probability that $(x)$ and $(y)$ will die in the same calender year [10 marks]

(b) (40) and (60) are lives subject to a common shock with $\lambda = 0.01$. In the absence of the common shock, (40) and (60) have independent lifetimes both following DeMoivres Law with $\omega = 100$. Find (i) ${}_{10}p_{40}$ (ii) ${}_{10}p_{40:60}$ (iii) $\mathring{e}_{40}$ (iv) $\mathring{e}_{40:60}$. [10 marks]

# QUESTION THREE [20 Marks]

(a) For a population comprising of smokers and nonsmokers, you are given:

- Nonsmokers have a force of mortality equal to one-half that of smokers of equal age.
- For nonsmokers, $\ell_x = 75 - x, \quad 0 \leq x \leq 75$

– (65) is a non-smoker

– (55) is a smoker

– (55) and (65) are independent lives.

Calculate $\mathring{e}_{65:55}$ [10 marks]

(b) William, aged 75, and Laura, aged 80, are the guardians of a child. They take out a life assurance policy that provides a payment of £25,000 immediately when the second of them dies. Level annual premiums are payable in advance whilst the policy is in force.

**Basis**:
Mortality : PMA92C20 for William, PFA92C20 for Laura
Interest : 4% pa effective

Expenses : Initial : £250, Renewal : 5% of each premium, excluding the first.

(i) Calculate the annual gross premium, using the basis given above. [6 marks]

(ii) Calculate the gross premium prospective reserve just before the sixth premium is paid, using the basis given below, assuming that both William and Laura are still alive at that time. [4 marks]

# QUESTION FOUR [20 Marks]

(a) Smith and Jones are independent lives aged 75 and 80, respectively, and both have mortality that follows DeMoivre's Law with $\omega = 100$. Find $\mathring{e}_{\overline{75:80}}$. [10 marks]

(b) A life insurance company issues an annuity to a male, aged 68, and a female, aged 65. The annuity of £10,000 pa is payable annually in arrears and continues until both lives have died. The insurance company values this benefit using PMA92C20 mortality for the male life, PFA92C20 mortality for the female life and 6% pa interest.

(i) Calculate the expected present value of this annuity. [2 marks]

(ii) Derive an expression for the variance of the present value of this annuity in terms of appropriate single life and joint life assurance functions. [4 marks]

(iii) Let X be the present value of the insurer?s profit from this policy. If the insurance company charges a premium of £150,000 for this policy, calculate $P(X > 0)$. [4 marks]

# QUESTION FIVE [20 Marks]

(a) You are given (i) $T(x)$ and $T(y)$ are independent. (ii) $E[T(x)] = E[T(y)] = 5.0$, (iii) $Cov[T(xy), T(\overline{xy})] = 0.08$. Calculate $E[(T(xy))]$ [7 marks]

(b) A life office sells joint whole life assurances to male lives aged 60 and female lives aged 55 exact. The benefits, payable at the end of the year of death in each case, are £100,000 on the first death and £50,000 on the second death. Level premiums are paid annually in advance while the policy is in force. Calculate the annual premium payable.
Basis: PMA92C20/PFA92C20 mortality, 4% pa interest. Ignore expenses. [8 marks]

(c) For the policy in the previous question 5(b), calculate the prospective reserve just before the payment of the 10th annual premium, assuming both lives are still alive at that point, and using the same basis as was used to calculate the premium. [5 marks]