TOM MBOYA UNIVERSITY

THIRD YEAR FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF
EDUCATION

MMA 300:REAL ANALYSIS

INSTRUCTIONS:

1. This paper consists of FIVE questions

2. Attempt questions ONE and any other TWO questions.

3. Observe further instructions on the answer booklet.

SECTION A

QUESTION 1 **[30 Marks]**

(a) Define the following.
(i) Interior point of a set. **[1 mk]**
(ii) Boundary point of a set **[1 mk]**

(b) Let $\{x_n\}_{n=1}^{\infty} = n + \frac{1}{n}$. Find the lower and the upperbound of this sequence if it exist. **[3 mks]**

(c) State and prove the uniqueness property of limits. **[5 mks]**

(d) Show that the sequence $\{s_n\} = \frac{n+1}{n}$ is a Cauchy sequence. **[5 mks]**

(e) Use integral test to test the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ for convergence or divergence. **[5 mks]**

(f) Use $\epsilon - \delta$ definition of limits to show that $\lim_{x \to 3}(x^2 + 2x) = 15$ **[5 mks]**

(g) Show that the function $f(x) = x^2$ is uniformly continuous on the interval $[-1, 1]$. **[5 mks]**

QUESTION 2 **[20 Marks]**

(a) Prove that the sequence $\{x_n\}_{n=1}^{\infty} = (-1)^n$ diverges. **[5 mks]**

(b) Let s_n and t_n be sequences of real numbers which converges to s and t respectively. Show that

$$\lim_{n \to \infty} s_n t_n = st$$

. **[8 mks]**

(c) Show that a constant function $f(x) = k$ is Riemann integrable in the interval $[a, b]$ hence find the integral. **[7 mks]**

QUESTION 3 **[20 Marks]**

(a) Show that the function $f(x) = \begin{cases} x\sin\frac{1}{x}, & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$ is continuous at 0 **[5 mks]**

(b) Prove that every convergent sequence of real numbers is bounded. **[5 mks]**

(c) (i) Prove that a geometric series with ratio r diverges if $\mid r \mid \geq 1$ and if $0 < \mid r \mid < 1$ the series converges to the sum $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$. **[7 mks]**
(ii) Find the sum of the series $\sum_{n=0}^{\infty} \frac{3}{2^n}$. **[3 mks]**

QUESTION 4 **[20 Marks]**

(a) Use Cauchy criterion to show that the sequence $\{\frac{(-1)^n}{n}\}$ converges. **[6 mks]**

(b) State and prove the Cauchy ratio test for testing the convergence of series. **[14 mks]**

QUESTION 5 **[20 Marks]**

(a) Define the following.
(i) A monotonic sequence. **[1 mk]**
(ii) an open ball. **[1 mk]**
(iii) A countable set. **[1 mk]**

(b) Prove that arbitrary union of open sets in a metric space is open. **[6 mks]**

(c) Test for convergence or divergence of the series given by $\sum \frac{n^2}{3^n}$. **[4 mks]**

(d) Prove that the set of real numbers is uncountable. **[7 mks]**