

TOM MBOYA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE
DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
WITH I.T

MAC 201 : FINANCIAL MATHEMATICS I

Instructions:

- Answer QUESTION ONE (compulsory) and any other TWO.
- Proofs should be written carefully.
- Cheating is NOT allowed and will be harshly punished.
- Observe further instructions on the answer booklet.

QUESTION ONE [30 Marks]

- (a) An investor is considering making an investment and is deciding between two possible alternatives:

- A 6-month investment which can be purchased at a simple rate of discount of 3.15% p.a. and
- a bank deposit, for 6 months, offering an effective rate of interest of 3.35% p.a.

Determine which of these two alternatives offers the higher rate of return. [4 marks]

- (b) Given that $\delta(t) = 0.03t$ for $0 \leq t \leq 10$, calculate the value of $(\bar{I}a)_{\overline{10}|}$. [3 marks]

- (c) Calculate the present value of a series of 10 annual payments, where the first payment is £550 made in one year's time, and each payment is £100 higher than the previous one. Assume an effective rate of interest of 7.5% pa. [4 marks]

- (d) A loan of 50,000 is repayable by level annual payments at the end of each of the next 7 years. Interest is 6.8% pa effective for the first three years and 12.5% pa effective thereafter. Calculate the loan outstanding immediately after the second repayment. [4 marks]

- (e) Calculate the effective annual rate of interest corresponding to;

- (i) a nominal rate of interest of 11.3% p.a convertible half-yearly.
- (ii) a nominal rate of interest of 12.9% p.a convertible monthly.

[4 marks]

- (f) Calculate the present value of an annuity that pays £3000 pa monthly in arrears forever using an annual effective rate of interest of 6.7%. [3 marks]

- (g) A man borrows £6,500 to buy a car. He repays the loan by 24 monthly instalments of £368.75, payable in arrears. Calculate the APR on this transaction. [4 marks]

- (h) The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \leq t < 2 \\ 0.045 - 0.0025t & 2 \leq t < 10 \\ 0.02 & t \geq 10 \end{cases}$$

Calculate the accumulated amount at time $t = 9$ of an investment of £15,000 made at time $t = 1$. [4 marks]

QUESTION TWO [20 Marks]

- (a) Smith's grandchild will begin a four year college program in one year. Smith wishes to open a bank account with a single deposit today to provide some funding for the grandchild's college education. Smith would like the grandchild to be able to withdraw 1700 USD from the account each year for four years, with the first withdrawal taking place one year from now, and subsequent withdrawals each year after that. Smith would like the account balance to be 0 immediately after the final withdrawal is made four years from now. The account has an annual effective interest rate of 5.3% . Determine the amount of the deposit Smith makes today. [5 marks]

- (b) If $s_{\overline{n}|} = 90$ and $s_{\overline{2n}|} = 270$, find the values of

- (i) $(i + i)^n$,
- (ii) i ,
- (iii) $s_{\overline{3n}|}$.

[10 marks]

- (c) Erick deposits X into a savings account at time 0, which pays interest at a normal rate of i , compounded semi-annually. Mike deposits $2X$ into a different savings account at time 0, which pays simple interest at an annual rate of i . Erick and Mike earn the same amount of interest during the last 6 months of the 8th year. Calculate i . [5 marks]

QUESTION THREE [20 Marks]

- (a) If $s_{\overline{3n}|} = X$ and $s_{\overline{n}|} = Y$, express V^n in terms of X, Y and constants [5 marks]
- (b) At an annual effective interest rate of i , $i > 0$, both the following annuities have a present value of X :
- (i) a 20-year annuity-immediate with payments of 55
 - (ii) a 30 year annuity-immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years.

Calculate X

[7 marks]

- (c) The force of interest at time t is given by:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \leq 1 \\ 0.05t - 0.01 & 1 < t \leq 5 \\ 0.24 & t > 5 \end{cases}$$

- (i) Derive and simplify as far as possible expressions for $A(t)$, where $A(t)$ is the total accumulated value at time $t(> 0)$ of an investment of 1 at time 0.
- (ii) A continuous payment stream is received at a rate of $25e^{0.02t}$ units per annum between time 5 and time 10. Calculate the present value (at time 0) of this payment stream.

[8 marks]

QUESTION FOUR [20 Marks]

- (a) An insurance company borrows £50 million at an effective interest rate of 9.5% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrears for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7.5% per annum.
- (i) Calculate the discounted payback period for this investment.
- (ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project.

[9 marks]

- (b) Distinguish between the discounted payback period and the payback period of an investment project. Therefore, describe the disadvantages of using these two measures for determining whether to proceed with an investment project or not.

[5 marks]

- (c) A company has just bought an office block for £5 million, which it will rent out to a number of small businesses. The total rent for the first year will be £100,500, and this is expected to increase by 4.6% pa compound in each future year. The office block is expected to be sold after 20 years for £7.5m. Assuming that rent is paid in the middle of each year, calculate the yield the company will obtain on this investment.

[6 marks]

QUESTION FIVE [20 Marks]

- (a) An investor is to pay £80,000 for a property. The investor will then be entitled to receive rental payments at the end of each year for 99 years. The rental payment will be fixed for the first 33 years, increasing to double the original amount for the next 33 years, and three times the original amount for the remaining 33 years. The value of the property at the end of the 99 years is expected to be £1,500,000. Calculate the amount of the rent payable in the first year, if the investor expects to obtain a rate of return of 8% pa effective on the purchase. [5 marks]
- (b) (i) Assuming a rate of interest of 6.6% pa effective, calculate the present value as at 1 January 2020 of the following annuities, each with a term of 25 years:
- (I) an annuity payable annually, where the first payment is £3,400 made on 1 January 2021, and payments increase by £550 on each subsequent 1 January.
 - (II) an annuity as in (i), but only 10 increases are to be made, the annuity then remaining level for the remainder of the term.
- [5 marks]
- (ii) An investor is to receive a special annual annuity for a term of 10 years in which payments are increased by 5% compound each year to allow for inflation. The first payment is to be £1,000 on 1 November 2021. Calculate the accumulated value of the annuity payments as at 31 October 2038 if the investor achieves an effective rate of return of 4.7% per half-year. [4 marks]
- (c) $s_{\overline{n}|} = 48.99$, $s_{\overline{n-2}|} = 36.34$, and $i > 0$, find i . [6 marks]