TOM MBOYA UNIVERSITY

UNIVERSITY EXAMINATION

SEMESTER 1, 2023/2024

UNIT CODE: MMA200 UNIT TITLE: CALCULUS II

TIME: 3HOURS

INSTRUCTION: ANSWER QUESTION ONE (COMPULSORY AND ANY OTHER TWO)

QUESTION ONE (30MKS)

a) Evaluate $\int_0^{\pi/4} 4\,Cos^2 3\theta\, d\theta$ (4mks)

b) Find mean value $\bar{y}$ of the function $y = f(x) = 3\sqrt{x}\ ,\ 0 \le x \le 4$ (4mks)

c) Show why $\int_{-1}^{1} \frac{1}{x^2}\, dx \neq -2$ (4mks)

d) Using the Fundamental theorem of integration, evaluate:

i) $\int_1^8 \left(\frac{1}{\sqrt[3]{x}} - \frac{5}{x}\right) dx$ (4mks)

ii) $\int_0^{\pi/2} Sin(3x)dx$ (4mks)

e)i) Find volume of solid of revolution formed by one revolution about *x-axis* of area enclosed by curve $y = 2x^3 + 3\ ,\ 0 \le x \le 2$ (5mks)

ii)Evaluate by algebraic substitution $\int \frac{2e^t}{\sqrt{e^t+4}}\, dt$ (5mks)

QUESTION TWO (20MKS)

a) With your knowledge of improper integrals, determine whether $\int_{-\infty}^{\infty} x\, Sin\, x^2 dx$ is convergent or divergent (8mks)

b) Evaluate $\int Sin^2 x\, dx$ hence solve $\int_0^{\pi/3} 3\, Sin^2(3x)\, dx$ (8mks)

c) Solve by trigonometric substitution $\int 2sin\, 3x \sin\ x\, dx$ (4mks)

QUESTION THREE(20MKS)

a) Using the method of integration by parts, solve $\int_0^{\pi/2} x\, Sin(2x)dx$ (6mks)

b) Evaluate $\int \frac{5}{16+9\theta^2}\, d\theta$ (4mks)

c) Test whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is convergent or divergent (4mks)

d) Solve the following using any suitable method $\int_0^{\pi/2} 3\, Sin^4\, \theta\, Cos\, \theta\, d\theta$ (6mks)

QUESTION FOUR (20MKS)

a) Given that $f(x) = x^4 - 2x^2$, $-2 \leq x \leq 2$. Determine whether Rolle's theorem can be applied and if so, find all values of *c* in the interval that satisfy the theorem's conclusion. (6mks)

b) Work out the following integral using any suitable method

$$\int_0^2 \frac{1}{4+x^2}\, dx$$ (6mks)

c) Using the method of integration by partial fractions, evaluate $\int \frac{4(x-4)}{x^2-2x-3}\, dx$ (8mks)

QUESTION FIVE(20MKS)

a) i) By taking a case of integrals of the form $\int Cos^m x\, Sin^n x\, dx$, where m is odd and *n* is even or vice versa, but not both even and not both odd, evaluate $\int Sin^3\theta\, d\theta$ (6mks)

ii) From the solution in (i) above, give a general expression for $\int Sin^3(n\theta)d\theta$ (2mks)

iii) With the knowledge of the integral of $\int Sin^3(n\theta)d\theta$ evaluate $\int_0^{\pi/3} 5\, Sin^3(4\theta)d\theta$ (4mks)

b) Determine the area between the curve $y = x^3 - 2x^2 - 8x$ and *x*-axis (8mks)